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Modelling of unsteady combustion regimes for polydisperse fuels—II. Parametrically controlled combustion

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Abstract—Results of computer calculations of essentially non-linear periodic combustion regimes originating far from the stability boundary are presented. The influence of the parametrical modulation of relevant processing parameters upon the neutral stability and upon the properties of oscillating combustion regimes is investigated. Modulation characteristics which cause either stabilization or destabilization of combustion are determined. Harmonic, ultra- and subharmonic frequency locking phenomena appearing in new regions of instability of the parametrically influenced combustion regimes are revealed. Quasiperiodic combustion states forming outside the regions of frequency locking are studied.

1. INTRODUCTION

CONCLUSIONS reached in the preceding part of this paper [1] constitute a sufficient set for determining the characteristics of auto-oscillating combustion regimes and for demonstrating their effectiveness as means of the intensification of technological processes.

A set of equations governing combustion of particulate fuels and comprising the heat and mass balance equations and the kinetic equation for the particle size distribution have been formulated in ref. [1]. In the same paper the system of functional integro-differential equations for the dimensionless temperature and the oxidant concentration have been derived as follows:

$$\left[1+\frac{H}{X}\int_0^\infty\left(\int_s^\infty g(x)F(x)(b_s/b-1)\,\mathrm{d}x\right)r^3(s)\,\mathrm{d}s\right]$$
$$\times\frac{b}{b_s}\frac{\mathrm{d}y}{\mathrm{d}t_*}+St+(St+St_*)y-\frac{4\pi k_1WB^2b_s^2b}{\rho_sc_sT_0u_s}$$
$$\times\int_0^\infty\left(\int_s^\infty\frac{g(x)F(x)\,\mathrm{d}x}{b(t_*+s-x)}\right)F(s)r^2(s)\,\mathrm{d}s=0, \quad (1)$$

$$\frac{b}{b_s}\frac{\mathrm{d}z}{\mathrm{d}t_*}-St_mG(v)+4\pi k_1WB^2b_s^2b$$
$$\times\int_0^\infty\left(\int_0^\infty\frac{g(x)F(x)\,\mathrm{d}x}{b(t_*+s-x)}\right)F(s)r^2(s)\,\mathrm{d}s=0, \quad (2)$$

where the following parameters have been introduced

$$v=(C-C_0)/C_0, \quad u=(T-T_0)/T_0$$
$$u_*=(T_*-T_0)/T_0, \quad y=(u-u_s)/u_s$$
$$z=(v-v_s)/v_s, \quad B=(b_s^4g_0F_0^4)^{-1/5}$$
$$St=\alpha(u_s-u_*)B/\rho_sc_su_s, \quad St_*=\alpha u_*B/\rho_sc_su_s$$
$$St_m=v_sBG_s/C_0, \quad t_*=\frac{1}{Bb_s}\int_0^t b\,\mathrm{d}t$$
$$s=\frac{1}{Bb_s}\int_0^r\frac{\mathrm{d}r}{F}, \quad H=\frac{4\pi(\rho_2c_2-\rho_1c_1)B^2b_sX}{3V\rho_sc_s}$$
$$X=\int_0^\infty\left(\int_s^\infty g(x)F(x)\,\mathrm{d}x\right)r^3(s)\,\mathrm{d}s. \quad (3)$$

Far from the stability boundary, the theory developed in ref. [1] for slightly non-linear auto-oscillating regimes becomes incorrect. Without going into particulars, typical dependencies will be given here for the amplitude and frequency of essentially non-linear oscillations forming deep in the instability region (see Figs. 1 and 2) on the supercriticality $(R_u-R_u^0)/R_u^0$. These curves have been obtained by numerical solving equations (1) and (2) by the Eitken–Steffensen iterative method [2]. It is worth noting that when the representative point in the parametric space (see Figs. 1 and 2 from ref. [1]) penetrates deep into the region of instability, the frequency of auto-oscillations tends to zero. In such a case, periodic regimes will be rarely observed in practice, since their period becomes infinite. It should also be noted that the numerical analysis gives an opportunity for determining the limits of correctness of the description of auto-oscillating combustion processes in the slightly non-linear approximation (compare Figs. 1 and 2 from the present paper and Fig. 6 from ref. [1]).

2. THE INFLUENCE OF MODULATION ON THE NEUTRAL STABILITY

A useful aspect of the theory developed is the presence of self-frequencies of oscillations in the system under study, which makes possible various phenom-

NOMENCLATURE

c, c_1, c_2	thermal capacity of mixture, gas and disperse phase, respectively
C	oxidant concentration
d_i	parameters introduced in equation (8)
D_i	coefficients introduced in equation (8)
f	particle size distribution
g	particle influx rate
h_i	parameters introduced in equation (8)
G	oxidant mass flux
k_1	coefficient defined by chemical reaction
k_2	stoichiometric coefficient
r	particle radius
s	dimensionless particle radius
St	heat transfer Stanton number
St_1	modulation amplitude
St_m	mass transfer Stanton number
St_{m1}	modulation amplitude
t	time
t_*	dimensionless time
T	temperature
u	dimensionless temperature
U_i	periodic functions specifying parametric modulation
v	dimensionless oxidant concentration
W	reaction heat.

Greek symbols

α	effective heat transfer coefficient
ρ, ρ_1, ρ_2	density of mixture, gas and dispersed phase, respectively
ω	oscillation frequency.

Subscript

s	steady combustion regime.

ena of parametric resonance. Some of them have been investigated earlier in refs. [3–6] for the crystallization from supersaturated or supercooled solutions and the boiling of highly superheated liquids. They play an important role in the optimization of technological processes. Analogous phenomena are to be expected when external parametrical excitation is applied to the furnaces with particulate fuels. In this connection the present paper analyses the influence of modulation of a number of parameters on the neutral stability of steady combustion states and on both the amplitude and frequency of auto-oscillating regimes.

Intensive auto-oscillations of the furnace temperature, sometimes capable of destroying combustion equipment, are undesirable. As has been shown in ref. [6], external periodic disturbances resulting in the modulation of relevant processing parameters are effective means of damping the instability. In the combustion system under study, the particle influx rate g and the heat and mass transfer Stanton numbers St and St_m may be regarded as controlled parameters. Thus, they can be represented in a modulated form being a result of the action of external periodic disturbances:

$$g = g_0[1+g_1U_1(t_*)], \quad St = St_0[1+St_1U_2(t_*)],$$
$$St_m = St_{m0}[1+St_{m1}U_3(t_*)]. \tag{4}$$

Here the index '0' refers to non-modulated quantities, and U_i ($i = 1, 2, 3$) are periodic functions.

The influence of periodic parametric disturbances on the neutral stability of steady combustion states can be analysed on the basis of the linearized equations (1) and (2) which now read

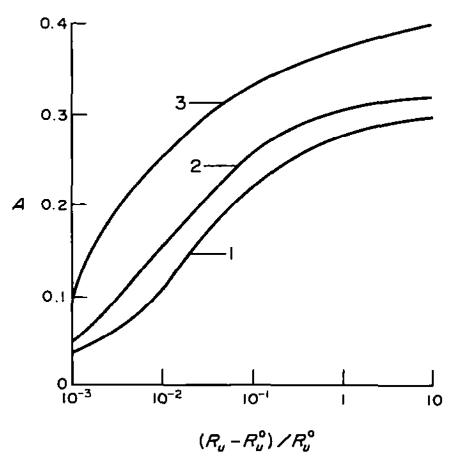

FIG. 1. The amplitude A of auto-oscillations depending on the supercriticality: (1) $St = 20$; (2) $St = 10$; (3) $St = 5$; $St_m = -0.01$, $St/St_* = -0.7$, $s_0 = s(r_0) = 10$, $g(r) = g_0P(r_0-r)$, P being the Heaviside function (see equation (15) from Part I of the paper), $R_v = -0.01$.

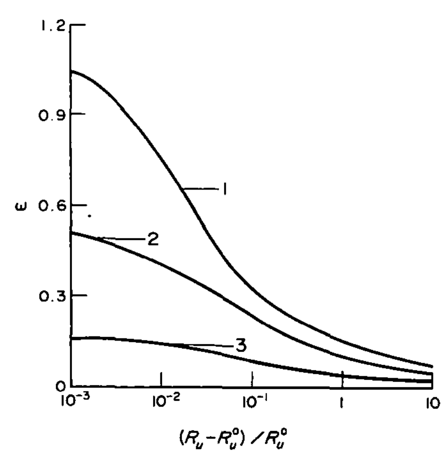

FIG. 2. The oscillation frequency ω depending on the supercriticality. Nomenclature is the same as in Fig. 1.

$$\frac{\mathrm{d}y}{\mathrm{d}t_*}+[(1-R_u)St_0(1+St_1U_2(t_*))+St_*]y$$
$$-R_vSt_0(1+St_1U_2(t_*))z+\frac{R_vSt_0}{Y}[1+St_1U_2(t_*)]$$
$$\times\int_0^\infty\left(\int_s^\infty g_0(x)[1+g_1U_1(t_*+s-x)]\right.$$
$$\left.\times F(x)y(t_*+s-x)\,\mathrm{d}x\right)F(s)r^2(s)\,\mathrm{d}s+\frac{R_vSt_0}{Y}$$
$$\times[1+St_1U_2(t_*)]\int_0^\infty\left(\int_0^\infty g_0(x)[1+g_1U_1(t_*+s-x)]\right.$$
$$\left.\times F(x)z(t_*+s-x)\right)F(s)r^2(s)\,\mathrm{d}s=0, \qquad (5)$$

$$\frac{\mathrm{d}z}{\mathrm{d}t_*}+[R_vSt_{m0}(1+St_{m1}U_3(t_*))-G_v]z+St_{m0}$$
$$\times[1+St_{m1}U_3(t_*)]R_uy-\frac{R_uSt_{m0}}{Y}[1+St_{m1}U_3(t_*)]$$
$$\times\int_0^\infty\left(\int_s^\infty g_0(x)[1+g_1U_1(t_*+s-x)]F(x)\right.$$
$$\left.\times y(t_*+s-x)\,\mathrm{d}x\right)F(s)r^2(s)\,\mathrm{d}s-\frac{R_vSt_{m0}}{Y}$$
$$\times[1+St_{m1}U_3(t_*)]\int_0^\infty\left(\int_s^\infty g_0(x)[1+g_1U_1(t_*+s-x)]\right.$$
$$\left.\times F(x)z(t_*+s-x)\,\mathrm{d}x\right)F(s)r^2(s)\,\mathrm{d}s=0. \qquad (6)$$

Periodic solutions of equations (5) and (6) exhibit a new neutral stability surface of steady combustion regimes. We shall begin analysis of equations (5) and (6) under an assumption that the modulation is of the most simple right-angled stepped type, that is, the functions U_i are to be represented in the following way

$$U_i(t_*)=\begin{cases}1, & 2\pi n/\omega<t_*<\pi(2n+1)/\omega,\\ -1, & \pi(2n-1)/\omega<t_*<2\pi n/\omega,\end{cases} \qquad (7)$$

where n is an integer number [7]. A useful aspect of this type of modulation consists in the simplicity of its practical realization.

Let the general solution of equations (5) and (6) in the interval $(-\pi/\omega,\pi/\omega)$ be expressed in the form

$$y_1(t_*)=\mathrm{e}^{-d_1t_*}(D_1\sin(h_1t_*)+D_2\cos(h_1t_*)),\quad -\pi/\omega<t_*<0,$$
$$y_2(t_*)=\mathrm{e}^{-d_2t_*}(D_3\sin(h_2t_*)+D_4\cos(h_2t_*)),\quad 0<t_*<\pi/\omega,$$
$$z_1(t_*)=\mathrm{e}^{-d_3t_*}(D_5\sin(h_3t_*)+D_6\cos(h_3t_*)),\quad -\pi/\omega<t_*<0,$$
$$z_2(t_*)=\mathrm{e}^{-d_4t_*}(D_7\sin(h_4t_*)+D_8\cos(h_4t_*)),\quad 0<t_*<\pi/\omega. \qquad (8)$$

Continuity conditions for y and z at the points $-\pi/\omega$, 0, π/ω are as follows:

$$y_1(0)=y_2(0),\quad \dot{y}_1(0)=\dot{y}_2(0)$$
$$y_2(\pi/\omega)=\mu y_1(-\pi/\omega),\quad \dot{y}_2(\pi/\omega)=\mu\dot{y}_1(-\pi/\omega)$$
$$z_1(0)=z_2(0),\quad \dot{z}_1(0)=\dot{z}_2(0)$$
$$z_2(\pi/\omega)=\kappa z_1(-\pi/\omega),\quad \dot{z}_2(\pi/\omega)=\kappa\dot{z}_1(-\pi/\omega). \qquad (9)$$

Here μ and κ are rational parameters, and the dot over a symbol denotes differentiation with respect to time. Substituting solutions (8) into the continuity conditions (9) gives a set of linear algebraic equations that couple the coefficients from D_1 to D_8. After a little algebra one can find characteristic determinants of this system yielding the following two rational equations

$$\cos(h_1\pi/\omega)\cos(h_2\pi/\omega)-\frac{(d_1-d_2)^2+h_1^2+h_2^2}{2h_1h_2}\times\sin(h_1\pi/\omega)\sin(h_2\pi/\omega)=\mu\cosh((d_1+d_2)\pi/\omega), \qquad (10)$$

$$\cos(h_3\pi/\omega)\cos(h_4\pi/\omega)-\frac{(d_3-d_4)^2+h_3^2+h_4^2}{2h_3h_4}\times\sin(h_3\pi/\omega)\sin(h_4\pi/\omega)=\kappa\cosh((d_3+d_4)\pi/\omega). \qquad (11)$$

When $|\mu|=|\kappa|=1$, equations (10) and (11) give periodic solutions corresponding to the neutral stability surface. They are complemented by a set of equations obtained by substituting equations (8) into equations (5) and (6). The latter two, together with equations (10) and (11), form a set of non-linear algebraic equations determining the neutral stability surface influenced by parametric modulation. This system has been analysed numerically. From Fig. 3 displaying results of the application of the modulation of the particle influx rate one can learn that parametric modulation can cause both stabilization and destabilization of steady combustion states. This depends on the characteristics of modulation. The

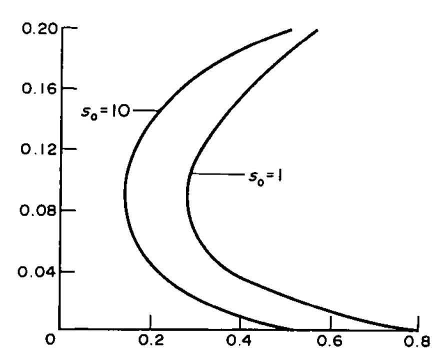

FIG. 3. The region of stabilization (to the right of the curve) and destabilization (to the left of the curve) for the stepped modulation of the particle influx rate, $St/St_*=-0.7$, $R_v=-0.01$, $St_m=-0.01$.

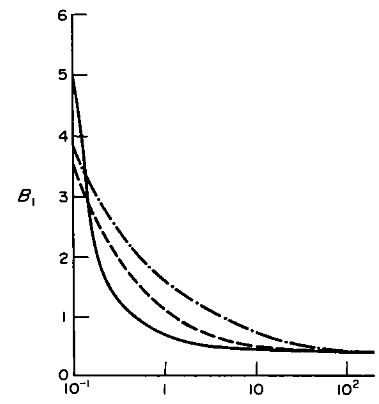

FIG. 4. Displacement of the neutral stability curves for the stepped modulation of the particle influx rate, $s_0 = 10$, $St/St_* = -0.7$, $R_v = -0.01$, $St_m = -0.01$; solid curve, $g_1 = 0$; dashed curve, $g_1 = 0.2$; dashed–dotted curve, $g_1 = 0.5$.

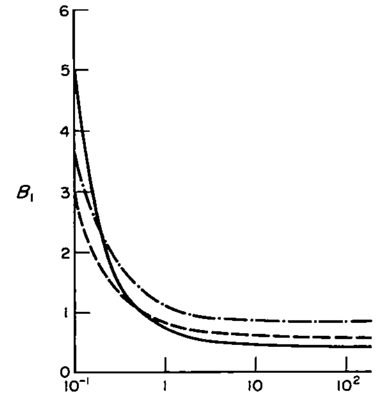

FIG. 6. Displacement of the neutral stability curves for the harmonic modulation of the particle influx rate, $s_0 = 10$, $St/St_* = -0.7$, $R_v = -0.01$, $St_m = -0.1$; solid curve, $g_1 = 0$; dashed curve, $g_1 = 0.2$; dashed–dotted curve, $g_1 = 0.5$.

corresponding displacement of the neutral stability curves is shown in Fig. 4. To the left of the point of intersection of the new neutral stability curves with the original ones the region of stability becomes narrow, and to the right it broadens. Similar results have been obtained for the modulation of heat and mass transfer Stanton numbers.

Educated by the stepped modulation applied right above, we can pass over to another type of external oscillating disturbance. Consider, by way of example, a harmonic modulation which provides an equality of mean effective influences (through a semiperiod) to those for the stepped modulation, i.e. $U_1(t_*) = \pi/2 \sin \omega t_*$. In such a case equations (5) and (6) can be solved numerically. The new neutral stability surface is determined by the periodic solutions of equations (5) and (6). The stability region corresponds to solutions of equations (5) and (6) vanishing in due time, and the instability region—to increasing ones. Figures 5 and 6 show that the transition to the harmonic modulation changes the picture of stability break only quantitatively.

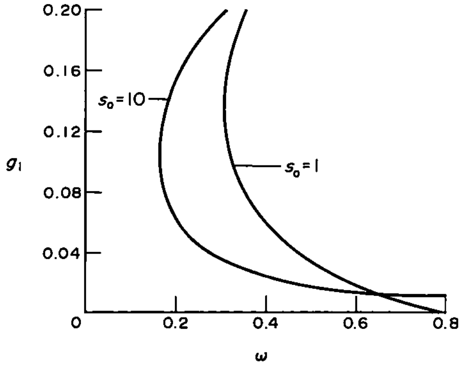

FIG. 5. The region of stabilization (to the right of the curve) and destabilization (to the left of the curve) for the harmonic modulation of the particle influx rate, $St/St_* = -0.7$, $R_v = -0.01$, $St_m = -0.01$.

3. FREQUENCY LOCKING AND QUASIPERIODIC OSCILLATING REGIMES

Investigation of the instability regions for parametrically stabilized combustion regimes has been conducted numerically by the Eitken and Steffensen iterative procedure on the basis of non-linear equations (1) and (2). A phenomenon of frequency locking has been revealed: under the influence of the periodic parameters the frequency of auto-oscillations coincides with the external one in some region being sufficiently close to the frequency of the self-oscillations (harmonic frequency locking). This phenomenon takes place if the 'internal' frequency is close enough to the external one and if the modulation amplitude is sufficiently large. The frequency locking has been revealed also at that case when the ratio of frequencies of auto-oscillations and external periodic influences is close to an integer number differing from unit. In such a case the frequency of the self-oscillations is captured by the frequency being the integer number of times smaller or greater than the external frequency—the so-called sub- and ultraharmonic frequency locking.

The regions of frequency locking are shown in Fig. 7. As distinct from the continuous crystallization from supersaturated solutions and supercooled melts [3, 5, 6] when ultra- or subharmonic frequency locking takes place at arbitrarily small modulation amplitude, in the system under study these phenomena are inherent to the basic harmonic only, the divisible frequencies being characterized by some critical amplitude which corresponds to the onset of the frequency locking. Typical cycles of captured oscillations are displayed in Fig. 8.

If the frequency of an external periodic influence corresponds to the point lying outside the regions of frequency locking, the system generates almost periodic oscillations. Unlike harmonically, ultra- and subharmonically captured oscillations having the per-

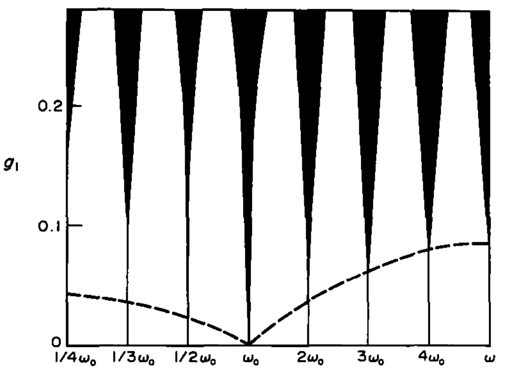

FIG. 7. The regions of frequency locking for the stepped modulation of the particle influx rate, $St = 1$, $s_0 = 10$, $St_m = -0.01$, $St/St_* = -0.7$, $R_u = 1.05R_u^0$, $R_v = -0.01$.

iod of basic harmonic, the integer number of times larger or smaller than the modulation period, quasiperiodic oscillations are not considered as periodic ones, their amplitude even at the developed stage undergoing a slow periodic change. The time of the whole periodic cycle in the oscillations shown in Fig. 9, form 5, 7..., 4, 1... and 12, 1... of the period of the parametric modulation.

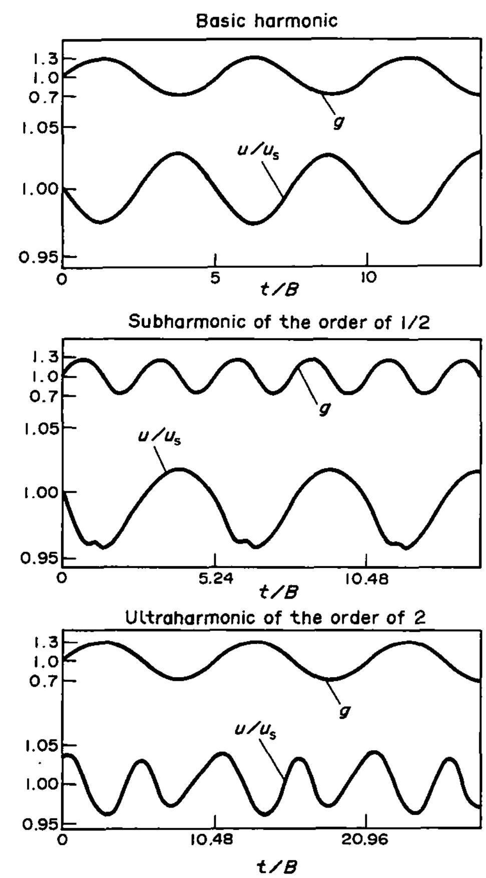

FIG. 8. The cycles of captured oscillations of relative temperature u/u_s for the modulation of the particle influx rate. Parameters are the same as in Fig. 7.

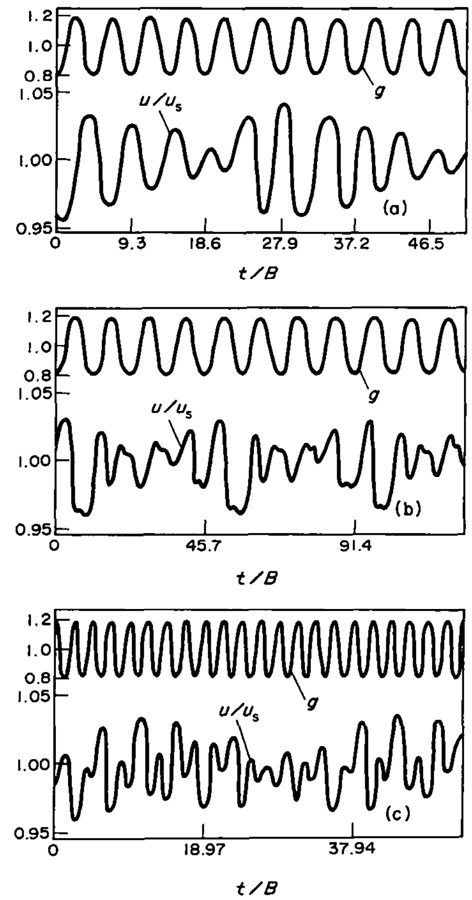

FIG. 9. Quasiperiodic oscillations of u/u_s originating from the basic harmonic (a), ultraharmonic of the 2nd order (b), subharmonic of the order of 1/2 (c). Parameters are the same as in Fig. 7.

4. CONCLUDING REMARKS

The theory presented provides one with a convenient tool in order to calculate beforehand essential properties of various possible regimes of the combustion of particulate solid and dropwise liquid propellants under different circumstances. First, it reveals conditions when steady combustion states lose their stability with respect to small fluctuations of the temperature and the oxidant concentration. Second, combustion processes on the threshold of instability have been studied in great detail. Depending on physical, chemical and regime parameters, the break of stability has been found to realize in either 'soft' or 'hard' manner. Thus one is able to come to a substantiated conclusion concerning the type of a periodic regime which will be established as a result of instability: the 'soft' type of instability gives rise to regular auto-oscillations, whereas the 'hard' one causes a chaotic pulsating regime. Third, the most beneficial in this respect is an analysis and ensuring applications of conceivable appropriate types of external modulation of relevant processing parameters, which are shown to be powerful means of an expedient action on combustion processes in order to change them in a desired direction. All this enables one to make a reasonable decision concerning a choice of optimal conditions of combustion by singling out a proper regime or a group of such regimes and to perform, on this basis, the thermal design of a furnace or another device with burning particles.

The significance of general methods developed is by no means exhausted by their application to the combustion problems considered. They could be well employed also while analysing numerous combustion processes in turbulent flows encountered in many technological schemes as well as to other systems with an exothermal chemical reaction whose rate is strongly dependent on the temperature and/or the reagent concentrations. Representative examples are offered by catalytic and stirred-tank chemical reactors. Moreover, the same reasoning is fairly applicable to systems whose operation is controlled by the kinetics of mass exchange between two phases caused by either a phase transition (bulk crystallization and boiling) or an immediate interaction of the phases (granulation).

As a final remark, note that unstable combustion states bear a direct relation to solving some ecological problems. For example, the output of nitrogen and sulphur oxides is dependent, first of all, upon the temperature level in a furnace. When the temperature oscillates, the kinetics of the oxides production can be thought of as being determined by some effective temperature value which happens sometimes, because of strong non-linearity of kinetic processes, to be much smaller than the actual mean temperature. This leads to a drastic decrease in the release of deleterious products and permits, thereby, the optimization of furnaces with respect to ecological requirements without negative influence on the operational characteristics by means of a corresponding choice of unsteady combustion regime. This problem may be regarded as one of the feasible ways of further investigations.

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